APPLICATION OF THE THEORY OF RANDOM PROCESSES TO COMBINED DESICCATION AND PNEUMATIC TRANSPORT OF SOLID PARTICLES WITH A TURBULENT GAS STREAM

J. Varchar'

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The method of mathematical statistics is used for analyzing the desiccation of solid particles during pneumatic transport, with the effect of turbulent gas fluctuations on the motion of particles taken into account.

1. Introduction. A theoretical analysis of the desiccation of solid particles in a suspended state shows that two characteristic cases must be distinguished here:

- a. The turbulent fluctuations are rapidly decaying and, because of the small size of particles, do not affect the motion of the latter.
- b. The size of particles is presumably such that turbulent gas fluctuations affect their motion significantly. Owing to the random turbulence in the gas stream, this process mode must be analyzed by methods used in the theory of random processes.

In all the technical literature known to the author these problems have not been discussed thoroughly enough.

Some studies have been published dealing with the pneumatic transport of particles by a turbulent gas stream, but without resorting to mathematical statistics as a method of analysis. Of particular interest among these studies is the one made by D. B. Spalding [4].

Certain general mathematical techniques employed by Spalding will be helpful in a statistical interpretation of the given physical phenomenon.

2. Physical Model. We consider a monodisperse system of solid particles in an unbounded turbulent gas stream. The moist particles and the turbulent gas are engaged in the desiccation process, i.e., in a transfer of heat and mass. The moisture content in the particles u is a function of time τ and of the space coordinates; the location of a particle is defined by the radius-vector **r**. Inasmuch as a turbulent gas stream transporting such particles is a random phenomenon, their moisture content u is obviously a random quantity and a function of the random variable $T = \{\mathbf{r}, \tau\}$ (**r** denoting the radius-vector of an arbitrary point in the three-dimensional Euclidean space, and τ denoting time).

In this case the problem can be formulated in terms of the probability density f(u, T) of moisture contents u in particles, i.e.,

$$P(u < \eta(T) < u - du) = f(u, T) du, \tag{1}$$

where f(u, T)du is the probability that the specific moisture content $\eta(T)$ in particles of a volume element δV containing the point **r** lies within the interval [u, u + du]. The volume element δV will be assumed sufficiently large to contain a definite statistical amount of particles, and yet sufficiently small for the statistical tical properties of these particles to be uniform throughout.

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From this definition of f(u, T) follows, obviously, that f(u, T)du may be interpreted as the ratio of the mass of particles with a moisture content within the infinitesimal interval [u, u + du] (i.e., $d^2M_u^{u+du}$) to the total mass of all particles and all the gas within the given volume element:

$$f(u, T) du = \frac{d^2 M_u^{u+du}}{dM_s}.$$
(2)

From definition (2) follows that the probability density f(u, T) is a continuous function T and signifies the concentration of particles whose moisture content, referred to a unit mass of mixture and a function of the specific moisture content u as well as of the space-time variable T, lies within the interval [u, u + du].

3. Derivation of the Differential Equation Describing the Physical Process. We consider a turbulent gas stream which carries solid particles. We assume that a change in the specific moisture content in the particles of a heterogeneous gas—solid mixture is caused by:

a) mass transfer from the particles to the turbulent gas stream;

b) random motion of particles in the turbulent gas stream.

In order to derive the differential equation which will describe this physical process, we single out a volume element δV in space according to the definition just given. We then obtain the relations, one by one, which describe the physical processes involved.

<u>Mass Transfer from the Particles to the Turbulent Gas.</u> We consider a monodisperse system of solid particles carried by a turbulent gas stream, and let every particle be characterized by its mean-overthe-volume specific moisture content u.

The loss of mass of particles dm during an element of time, due to mass transfer to the ambient medium, is described by the equation

$$-dm = s\beta_x (x_M - x) d\tau, \tag{3}$$

where the specific surface of a system of particles is s (m^2/kg) . The desiccation rate during the first period is then

$$\frac{du}{d\tau} = -s\beta_x (x_M - x).$$

(4)

During the second period $du/d\tau$ depends on the thermal conductivity and the hygroconductivity of the dried particles, i.e., mainly on their internal state.

For a qualitative analysis, however, it suffices to describe the effect of internal conditions on the desiccation process by any known equation for the desiccation rate.

Starting out with the conventional Lykov equation [5] for the decreasing-rate period, we have the relative rate

$$\chi = \frac{(du/d\tau)}{(du/d\tau)_{\rm Max}},$$

with $(du/d\tau)_{max}$ denoting the desiccation rate during the constant-rate period.

According to [6], the relative desiccation rate χ is proportional to $(u-u_E)$. For this reason, it can be represented as a general function

$$\chi = \chi \left(u - u_{\rm E} \right). \tag{5}$$

During the first period of desiccation $\chi(u-u_E)$ is equal to unity.

On the basis of Eq. (5), we can now generalize the equation for the desiccation rate during the constant-rate period (4) and during the decreasing-rate period

$$\frac{du}{d\tau} = -s\beta_x \left(x_M - x\right) \chi \left(u - u_R\right).$$
(6)

On the assumption that the equilibrium moisture content in a material during desiccation varies within a narrow range only, one may regard this equilibrium moisture content as constant and, instead of Eq. (6), write

TABLE 1	. Prob	ability Dis	stribution]	Density of	the Rando	m Variabl	Ð					
				п					п			
	0,60	0,54	0,49	0,42	0,35	0,32	0,27	0,22	0,17	0,13	0,09	0.07
z, m			-	1-11 E					B ⁿ⁻ⁿ E		internet of a second seco	
	0,53	0,47	0,42	0,35	0, 28	0,23	0,20	0, 15	0,10	0,06	0,02	0
8 80,00 80,00 80,00 80,00 80,00 8	800000000	0 1,6910-1 5,5810-1 5,5810-1 2,30100 2,80100 2,80100 1,01.10-3 0 0	$\begin{array}{c} 0\\ 6, 30.10^{-2}\\ 1, 10.10^{-1}\\ 3, 92.10^{-1}\\ 9, 12.10^{-1}\\ 1, 872.10^{-1}\\ 1, 872.10^{-2}\\ 2, 54.10^{-2}\\ 5, 99.10^{-27}\\ 0\end{array}$	0 2,69.10-2 4,70.10-2 8,87.10-2 1,68.10-3 1,68.10-3 1,68.10-3 1,05.10-3 1,05.10-1 1,94.10-15 0	$\begin{array}{c} 0\\ 1,46\cdot10^{-2}\\ 2,56\cdot10^{-2}\\ 4,84\cdot10^{-2}\\ 9,19\cdot10^{-2}\\ 2,28\cdot10^{-1}\\ 6,48\cdot10^{-1}\\ 1,79\cdot10^{-19}\\ 1,79\cdot10^{-19}\\ 0 \end{array}$	0 1,09-10-2 3,62-10-2 5,06-10-1 5,06-10-1 5,70-10-1 9,95-10-1 9,95-10-1	6,11.10-8 2,01.10-8 3,822.10-8 3,44.10-1 4,30.10-1 6,44.10-1	0 3,19-10-3 7,58-10-3 1,05-10-2 2,00-10-2 4,91-10-3 6,09-10-1 3,55-10-4	0 1,40.10 ⁻³ 2,46.10 ⁻³ 4,64.10 ⁻³ 8,83.10 ⁻³ 8,83.10 ⁻³ 7,61.10 ⁻² 1,02.10 ⁻¹	0 5,71.10-4 1,88.10-3 3,58.10-3 8,87.10-3 3,12.10-3 3,12.10-3 7,60.10-1 7,60.10-1	0 1,16.10-4 3,384.10-4 7,30.10-4 1,30.10-4 1,30.10-4 6,66.10-3 6,70.10-3 6,70.10-3 0,69.10-1	0000000008

$$\frac{du}{d\tau} = -s\beta_x \left(x_M - x\right) \chi \left(u\right). \tag{7}$$

It follows from the physical aspects of desiccation that, apparently, the quantity $s\beta_X(x_M-x)$ depends only on the physical properties of the drying agent:

- a) the mass transfer coefficient β_X , referred to the difference between the moisture content levels in the drying agent, depends only on the hydrodynamics and the thermodynamics of flow around a given particle;
- b) the quantity x_M is proportional to the moisture content in the drying agent at the particle surface. During the first period of desiccation x_M is, in fact, equal to the moisture content in the drying agent at the surface of free liquid when the latter evaporates adiabatically. Then x is equal to the moisture content in the drying agent outside the diffusion boundary layer of a particle.

If the specific surface of a given material s is known, then the quantity

$$\Phi = s\beta_x \left(x_M - x \right) \tag{8}$$

may be regarded as a function of the physical properties of the turbulent gas which transports particles of that material.

It follows from the preceding analysis of the mass transfer process in the gas—solid system that the kinetics of this physical phenomenon can be described by the general relation

$$\frac{du}{d\tau} = -\Phi\chi(u). \tag{9}$$

The material balance in a volume element δV , namely the change of moisture content in particles it contains can, with the aid of Eq. (9) and application of variational calculus, be written as

$$\left(\frac{\partial f\left(u; \mathbf{r}, \tau\right)}{\partial \tau}\right)_{P} = \Phi \frac{\partial}{\partial u} \left[f\left(u; \mathbf{r}, \tau\right) \chi\left(u\right)\right], \tag{10}$$

where the subscript P denotes a change in probability density with time and due to mass transfer from the particles to the turbulent gas stream.

<u>Turbulent Stream of Particles</u>. The time variation of absolute probability $f(u; \mathbf{r}, \tau)$ in the given volume element δV is also affected by the turbulent stream of particles which bounds this volume element at the control surface.

The mass flux of particles carried by the turbulent gas stream, when treated as a vector, can be resolved (just as the velocity of the turbulent gas stream) into a deterministic and a stochastic component. The fundamental deterministic component of the mass flux vector will be denoted by $j_M(r)$. On it we super-impose the stochastic component, which, for simplicity, will be defined in terms of the turbulent diffusivity ε of solid particles through the gas. The mass flux of particles with a moisture content within the [u, u + du] interval due to turbulent diffusion is proportional to the product of their concentration gradient (according to definition, the density of absolute probabilities is proportional to its gradient $\nabla f(u; r, \tau)du$) and the turbulent diffusivity ε , namely:

 $\rho_s \in \nabla f(u; \mathbf{r}, \tau) du.$

The total mass flux of particles is then equal to the sum of its deterministic and stochastic components:

$$\left[-\mathbf{j}_{\mathcal{M}}(\mathbf{r})f(u;\,\mathbf{r},\,\tau)+\rho_{s}\varepsilon\nabla f(u;\,\mathbf{r},\,\tau)\right]du.$$
(11)

The time variation in the density of absolute probabilities $f(u; \mathbf{r}, \tau)$ is, according to the Gauss-Ostrogradskii postulate, determined from the following differential equation:

$$\left(\frac{\partial f(u; \mathbf{r}, \tau)}{\partial \tau}\right)_{T} = -\frac{1}{\rho_{s}} \nabla \left(\mathbf{j}_{M}(\mathbf{r}) f(u; \mathbf{r}, \tau)\right) + \frac{1}{\rho_{s}} \nabla \left(\epsilon \rho_{s} \nabla f(u; \mathbf{r}, \tau)\right),$$
(12)

with the subscript T denoting the time variation of probability density due to a turbulent stream of particles.

The total time variation in the density of absolute probabilities is determined from Eqs. (10) and (12):

$$\frac{\partial f(u; \mathbf{r}, \tau)}{\partial \tau} = \Phi \frac{\partial \left[f(u; \mathbf{r}, \tau) \chi(u) \right]}{\partial u} - \frac{1}{\rho_s} \nabla \left(\mathbf{j}_M(\mathbf{r}) f(u; \mathbf{r}, \tau) + \frac{1}{\rho_s} \nabla \left(\varepsilon \rho_s \nabla f(u; \mathbf{r}, \tau) \right) \right). \tag{13}$$

Equation (13) may be interpreted as expressing the law of probability conservation for a random function $u(\mathbf{r}, \tau)$.

If the random process is stationary, then the density of absolute probabilities is independent of time τ and the partial derivative with respect to time is this equal to zero; Eq. (13) becomes then

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$$-\Phi \frac{\partial}{\partial u} \left[f(u; \mathbf{r}) \chi(u) \right] = -\frac{1}{\rho_s} \nabla \left(\mathbf{j}_M(\mathbf{r}) f(u, \mathbf{r}) \right) + \frac{1}{\rho_s} \nabla \left(\epsilon \rho_s \nabla f(u, \mathbf{r}) \right).$$
(14)

This equation can be simplified by the following change of variables:

$$f(u, \mathbf{r}) \chi(u) = \varphi(U, \mathbf{r}), \qquad (15)$$

$$\frac{d(u)}{\chi(u)} = -dU.$$
(16)

With (15) and (16) inserted into (14), a few necessary transformations yield

$$\Phi \rho_s \frac{\partial \varphi (U, \mathbf{r})}{\partial u} = -\nabla \left(\mathbf{j}_M (\mathbf{r}) \varphi (U, \mathbf{r}) \right) + \nabla \left(\epsilon \rho_s \nabla \varphi (U, \mathbf{r}) \right).$$
(17)

Transformation of the Differential Equation for the One-Dimensional Case. The differential equation (17), which describes the desiccation of solid particles carried by a turbulent gas stream, can be simplified under certain assumptions.

Let us consider; for example, the one-dimensional case in the direction of the x_3 -axis, which for clarity, will be called the z-axis. This axis is collinear with but opposite to the gravitation vector. In this case Eq. (17) will become

$$\Phi(z)\rho_s \frac{\partial \varphi(U,z)}{\partial U} = -\frac{\partial}{\partial z} (j_M(z)\varphi(U,z)) + \frac{\partial}{\partial z} (\varepsilon \rho_s \frac{\partial}{\partial z} \varphi(U,z)).$$
(18)

For an analytical solution to Eq. (18) we make further simplifications. Function $\Phi(z)$ defined by Eq. (8) depends generally on the location of the material in the duct. The difference between the moisture content levels in the drying agent at a particle surface x_M and in the ambient stream x, respectively, is generally a function of the z-coordinate and can, within a certain approximation, be replaced by its arithmetic mean. If we further assume that for very small particles Re $\rightarrow 0$ and Sh $\rightarrow 2$, then also $\beta_X = \text{const}$ at a constant molecular diffusivity. Based on these assumptions, we may approximately let

$$\Phi(z) = \Phi = \text{const.} \tag{19}$$

In the general case vector j_{M} is a function of the position vector **r**; in the one-dimensional case j_{M} is a function of the z-coordinate only. In the first approximation, however, j_{M} in (18) may be regarded as the average mass flux of material; average of its values at the desiccator entrance and exit

$$\Phi \rho_s \frac{\partial \varphi \left(U, z \right)}{\partial U} = \frac{\partial}{\partial z} \left(\epsilon \rho_s \frac{\partial}{\partial z} \Phi \left(U, z \right) \right) - j_M \frac{\partial \varphi \left(U, z \right)}{\partial z}.$$
(20)

This equation can be further simplified by an introduction of new variables:

$$\epsilon \rho_s d\xi = dz,$$
 (21)

$$\varphi(U, \xi) = \psi(U, \xi) \exp\left(\frac{1}{2} j_M \xi\right).$$
(22)

After transformations, we have

$$\Phi \varepsilon \rho_s^2 \frac{\partial \psi(U, \xi)}{\partial U} = \frac{\partial^2 \psi(U, \xi)}{\partial \xi^2} - \frac{j_M^2}{4} \psi(U, \xi) .$$
(23)

The second-order partial differential equation (23) is a nonhomogeneous one, but with the new dependent variable

 $\frac{1}{\alpha} = \Phi \epsilon \rho_s^2$,

$$\psi(U, \xi) = \omega(U, \xi) \exp\left(\frac{\alpha U j_M^2}{4}\right), \qquad (24)$$

where

it can be reduced to a homogeneous one

$$\frac{\partial \omega (U, \xi)}{\partial U} = \alpha \frac{\partial^2 \omega (U, \xi)}{\partial \xi^2}.$$
 (26)

This equation is analogous to the equation of transient heat conduction

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2}$$
(27)

and can be solved analytically for given initial and boundary conditions.

Conditions for a Unique Solution. We will now make the following assumption.

At z = 0 the moisture content is almost the same and equal to u in all particles. We have then for z = 0

$$\left. f(u, z) \right|_{z=0} = f(u, 0) = f(u) = \begin{cases} \infty & \text{for } u = u_A, \\ 0 & \text{for } u \neq u_A. \end{cases}$$
(28)

Furthermore, the moisture content in particles at every $z \neq 0$ can be

$$u_{\rm E} < u < u_A. \tag{29}$$

The constraints on the density of the probability distribution f(u, z) which would correspond to inequality (29) and to the general properties of function f(u, z) can be formulated as

$$f(u_A, z) = 0, \tag{30}$$

$$\int_{u_{A}}^{u_{E}} f(u, z) \, du = 1.$$
(31)

Relations (28), (30), and (31), which define the uniqueness conditions for the solution to Eq. (26), will be expressed now in new independent variables U, ξ and a dependent variable $\omega(U, \xi)$. Condition (28) becomes then

$$\lim_{\xi \to \frac{I_M}{2}} \omega(U, \xi) = \begin{cases} \infty; \ U = U_A = 0, \\ 0; \ U \neq 0 \end{cases}$$
(32)

and conditions (30), (31) become

$$\lim_{U \to 0} \omega(U, \xi) = 0, \tag{33}$$

(25)



Fig. 1. Probability density f(u, z) as a function of the moisture content u and of the z-coordinate.

$$-\int_{0}^{U_{\mathrm{E}}} \omega(U, \xi) \exp\left[-\left(\frac{\alpha U j_{M}^{2}}{4} - \frac{j_{M}\xi}{2}\right)\right] dU = 1, \qquad (34)$$

respectively.

The transformed differential equation (26) together with the transformed uniqueness conditions (32), (33), (34) determine, in the stationary case, the probability distribution density of moisture in particles, as a function of their distance from the duct entrance.

Analytical Solution. The absolute-probability distribution density of the random variable u(z) in the one-dimensional stationary case is determined (under certain simplifying assumptions made earlier) from the differential equation (26) with initial conditions (33), (34) and boundary conditions (32). Since the boundary condition (31) is not a continuous function, hence the most effective method of solving this problem mathematically will be the operational calculus according to Mikusinski which is based on convolution of the product of functions and is particularly convenient for problems of this kind [8].

The Mikusinski method of operational calculus will be applied here to the variable which is not continuous under the uniqueness conditions. In our problem, which involves definition (3), this variable is U.

The final solution to the problem can be obtained in the following form [7]:

$$\omega(U, \xi) = \lambda(\xi) \left\{ \frac{\sqrt{\alpha}(\xi - j_M/2)}{2\sqrt{\pi U^3}} \exp\left[-\frac{\alpha(\xi - j_M/2)^2}{4U}\right] \right\} h^{U_A},$$
(35)

with

$$\omega(u) = \text{const} \quad \frac{1}{u^{2/3}} \exp\left(-\frac{\text{const}}{u}\right),$$

for $\xi = \text{const}$ and with coefficient $\lambda(\xi)$ defined as

$$\lambda(\xi) = -\left[\int_{0}^{0} \left\{\frac{\sqrt{\alpha}(\xi - j_M/2)}{2\sqrt{\pi U^3}} \exp\left[-\frac{\alpha(\xi - j_M/2)^2}{4U}\right]\right\} \exp\left[-\left(\frac{\alpha U j_M^2}{4} - \frac{j_M \xi}{2}\right)\right] du\right]^{-1}.$$
(36)

If integral (36) can be evaluated, then the result will obviously be a special transcendental function. The quantity h^{UA} in Eq. (35) represents the Mikusinski shift operator.

4. Analysis of the Solution. The resulting Eq. (34) makes it feasible to calculate function $\omega(U, \xi)$, which is a transform of the absolute-probability distribution density of the random variable u(z) in the onedimensional stationary case. The form of function $\omega(U, \xi)$ according to Eq. (35) indicates that, while it does not correspond to the normal two-dimensional law, it can, on the basis of the definition of parameter $\lambda(\xi)$, be described by special transcendental functions. The resulting solution for $\omega(U, \xi)$ and the probability distribution density f(u, z), which has been specially calculated after an inverse transformation, yield specific estimates as to how random turbulent fluctuations in the stream of drying agent affect the uniformity of moisture content in solid particles carried by pneumatic transport. As an example, we have calculated functions (35) and (36) numerically on a model EVM-Z-23 computer with the aid of an inverse transformation, and also the probability distribution density of the random variable u (at various values of z) for the following parameters:

$$t_L = 150^{\circ} \text{ C}; \ |j_M| = 3.3 \text{ kg/m}^2 \cdot \text{sec}; \ \varepsilon = 0.412 \text{ m}^2/\text{sec};$$

 $\rho_s = 2.25 \cdot 10^3 \text{ kg/m}^3; \ \alpha = 0.046 \text{ m}^4 \cdot \text{sec}^2/\text{kg}^2;$
 $u_0 - u_F = 0.531 \text{ kg/kg}.$

Some results of this numerical evaluation are shown in Table 1. The calculated values of the probability distribution density of f(u) exceed 10^{29} , which makes their graphical presentation difficult. For this reason and for clarity, the trend of function f(z) is shown only schematically.

Thus, such a calculation based on the given simplifying assumptions yields data on the uniformity of desiccation, which are especially important in those cases where the subsequent technological processing requires that particles of the material be dried to a definite residual moisture level with rather little variance.

NOTATION

f(u, T)	is the probability density function;
j _M	is the deterministic component of the mass flux of particles vector;
m	is the mass flux of evaporating substance, kg/sec;
r	is the radius-vector;
S	is the specific surface of particles in dispersion, m ² /kg;
t	is the temperature, °C;
u	is the moisture content in the solid particles, kg/kg;
х	is the moisture content in the gas, kg/kg;
x _M	is the moisture content in the gas at the surface of particles, kg/kg;
Z	is the space coordinate, m;
М	is the mass, kg;
Р	is the probability;
т	is the space-time variable;
U	is a variable defined by Eq. (16);
V	is the volume, m ³ ;
α	is a coefficient defined by Eq. (25);
$\beta_{\mathbf{X}}$	is the mass transfer coefficient, kg/m ² · sec;
ε	is the turbulent diffusivity;
ξ	is a coordinate defined by Eq. (21);
λ	is a coefficient defined by Eq. (36);
ρ	is the density, kg/m^3 ;
τ	is time, sec;
ψ, ω, Φ, φ, χ	are functions defined by Eqs. (23), (26), (8), (15), (5);
Re	is the Reynolds number;
Sh	is the Sherwood number.

Subscripts

- A refers to the entrance conditions;
- M refers to the solid material;
- E refers to the equilibrium state;
- s refers to the dry state.

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